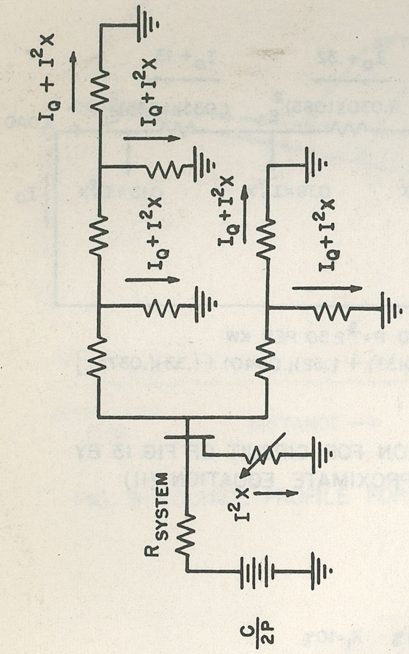


FIG. 19 D-C CIRCUIT FOR SOLUTION OF DISTRIBUTION PORTION OF POWER SYSTEM

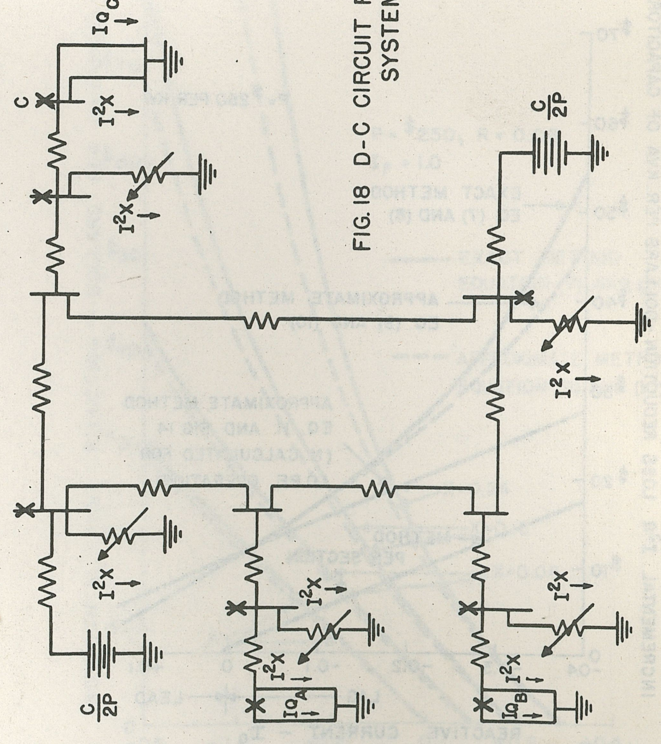
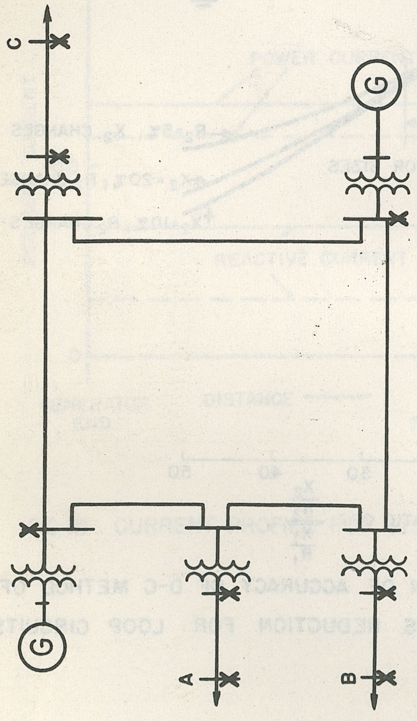


FIG. 18 D-C CIRCUIT FOR SOLUTION OF A POWER SYSTEM OF FIG. 17



X- POINTS WHERE VOLTAGE IS HELD CONSTANT

FIG. 17 ONE LINE DIAGRAM OF A POWER SYSTEM

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Conference Paper

D-C CIRCUIT GIVES EASY METHOD OF DETERMINING VALUE OF CAPACITORS IN REDUCING I^2R LOSSES

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55-711

D-C Circuit Gives Easy Method of Determining Value of Capacitors in Reducing I^2R Losses

R. A. Schmidt

The purpose of this paper is to present a method of determining the value of capacitors in reducing I^2R losses. This method treats the power system as a d-c circuit and determines the amount and location of shunt capacitors to give minimum cost.

The author believes that the method will prove to be useful not only in the distribution field but that it will also prove to be useful in determining the economic power factor for the different load areas of the complete power system.

Shunt capacitors perform many valuable functions in a power system. Capacitors can reduce the voltage drop and thereby increase the permissible loading on voltage limited circuits. Similarly by raising the power factor, capacitors can increase the load that may be carried by such thermally limited equipment as transformers and cables. Capacitors can also reduce I^2R losses. As a result of the many uses of capacitors, there are also many ways to evaluate capacitors. In this paper, only the ability of capacitors to reduce I^2R losses will be investigated.

It is realized that the determination of the amount of I^2R loss reduction produced by capacitors, does not necessarily show the full value of capacitors. Capacitors may have more value in improving voltage or in releasing system thermal capacity. However, the reduction of I^2R losses does set a floor under the value of capacitors; capacitors may be worth more, but never less. In many cases, sufficient capacitors can be justified solely on the basis of reduced I^2R losses to raise the power factor to a high value. In such cases, a study of voltage improvement and released thermal capacity may not be necessary.

The primary objective of this paper is to present a method, and not to draw conclusions. Two important facts, however, are often overlooked and will be pointed out here:

(1) Leading power factor operation does not necessarily mean increased system I^2R losses. As leading current may decrease the magnitude of power current in other parts of the system, it is necessary to look at the total current in order to determine the change in I^2R losses.

(2) It is necessary to include the effects of all system I^2R losses in order to get a true picture of capacitors and losses. Not only are the transmission, subtransmission and distribution losses important, but the losses in transformers and generators are also important.

SUMMARY

As the d-c circuit method of handling the capacitor and I^2R loss problem is new to most engineers, the purpose of the summary is to point out the areas that the paper will cover. The summary is too short to serve as an explanation of the method.

The object of the d-c circuit method is to determine the amount of capacitors that will result in minimum total cost of capacitors and I^2R losses. In order to obtain such an answer, the I^2R losses are computed on an incremental basis, as Kw of I^2R loss reduction per Kvar of capacitor, or dollars of I^2R loss reduction per Kvar of capacitor.

The mechanics of the method are outlined as follows: A convenient base Kva is chosen. Each generator is replaced by a d-c voltage and a per unit resistance representing its loss. All system elements, as transformers and lines, are replaced by their per unit resistances. All load points are grounded. The current magnitudes are then calculated or measured on a d-c board. The currents that flow in such a circuit are approximately equal to the reactive current that should flow in the a-c power system in order to obtain minimum cost. The reactive currents in the actual a-c system are then compared with the currents in the d-c circuit to determine how much kilovar correction is justified.

It will be shown that reactance affects the amount of I^2R loss reduction produced by capacitors. The method of treating reactance will be given in Section II of this paper.

This method assumes that a Kw of I^2R loss has a definite dollar cost. This paper will not attempt to say how a Kw of I^2R loss should be evaluated, but the three component parts of the cost of a kw of I^2R loss will be pointed out. One part is the demand charge for a Kw of generation due to the power plant investment[1]. A second part is the fuel or energy charge. The third part is the system demand charge due to the costs of lines and substations in the transmission and distribution areas.

The paper is divided into these three sections:

(1) In the first section, the voltage at the load bus is held constant. The load voltage is held constant by changing the sending end voltage. The assumption of constant load voltage means that the problem is simplified for two reasons. One reason is that the power and reactive requirements of the load do not change as capacitors are added. The second is that a constant Kw load and constant load voltage mean that the power component of current does not change when capacitors are added.

Using this assumption, capacitors reduce the voltage drop between the generator and the load end, and also reduce the I^2R losses. The reduced I^2R losses are due solely to lower reactive currents. The reduced I^2R losses are then evaluated on a dollar basis. A d-c circuit is used to make this evaluation. The reduced voltage drop is considered only as an extra benefit of capacitors; the reduced voltage drop is not evaluated on a dollar basis.

(2) In the second section, the voltage is held constant at both the load and generator ends when capacitors are added. To control voltage in this manner, the turns ratios of transformers and voltage regulators located between the generator and the load, must be changed. This method assumes that the original voltage conditions are satisfactory. When transformer taps are changed, both the power and reactive components of current are changed. A d-c circuit is then used to calculate the change in I^2R losses due to the change in both power and reactive components of current.

The accuracy of the d-c method is also discussed in this section. Nearly all errors in the d-c method tend to make the calculated I^2R loss reduction less than the actual I^2R loss reduction. For most power systems, the author estimates that the actual I^2R loss reduction will be 5-20 percent more than shown by the d-c circuit method. Further investigation is needed to determine whether this estimate is correct, especially for cases where the d-c method is used on a complete power system. The amount of error will naturally depend upon the characteristics of each power system.

(3) The third section discusses several points concerning the mechanics of obtaining an answer by a d-c circuit.

Use of Symbols

Each symbol will be defined as it is introduced. In addition, a list of symbols is included at the end of the text.

Section I. Capacitor Addition With Changing Sending End Voltage.

In this section the load voltage is assumed to be constant. When capacitors are added, the load voltage is held constant by lowering the sending end voltage. This assumption is made in order to simplify the problem; in Section II both the sending and receiving voltage will be held constant.

Solution of A Simple System

The method of using a d-c circuit will first be used to solve the simple circuit of Figure 1. The circuit of Figure 1 consists of one generator, one line, and one load. The load is 1000 kva at 0.8 P.F. (800 Kw and 600 Kvar). On a 1000 Kva base, the line has a 10% I^2R loss. The line also has reactance; but as the load voltage is held constant by changing the sending end voltage, the reactance does not affect the current or the I^2R loss. When capacitors are added, only the reactive current changes; the power current remains constant.

The reactive portion of the load can be supplied by shunt capacitors at the load, by the generator, or partly by the generator and partly by shunt capacitors. The desired answer is how many capacitors should be installed at the load in order to obtain minimum cost.

(A) Solution by Curves

The cost of I^2R losses may be expressed as:

(1) Cost of losses = $L = P(I_Q^2 \cdot R)$

Where all quantities are expressed in per unit of a given Kva base and
I_Q = the reactive current flowing in the line
R = line resistance
P = the dollar value of a Kw of I^2R loss

The loss value, P, may be either on an annual cost or on a capitalized cost basis. Although there is less possibility of error when economic studies are made on the basis of annual costs, 2, 3 capitalized costs will be used in the examples of this paper, because it seems to be the more common method.

The loss value, P, should include dollars for both the energy and the demand part of the cost of a Kw of I^2R loss. The dollar figure to be used for the loss value, P, will depend upon the practice of the individual company making the study.

The reactive current in the line, I_Q can be replaced by $(I_{Q_L} - I_C)$, where

I_{Q_L} = the load reactive current
I_C = the capacitor current

Equation (1) is now written as:

(2) $L = P\,(I_{Q_L} - I_C)^2\,R$

The added cost of reactive kva supplied at the load is:

(3) Added reactive cost = $K = (C_L - C_G) = CI_C$

Where the quantities are in per unit of the same kva base used for equations 1 and 2, and

C_G = Cost of a kva of reactive power supplied from the generator
C_L = Cost of a kva of reactive power supplied at the load by capacitors
C = Difference in cost of a kva of reactive power supplied at the load and a kva of reactive power supplied by the generator.

The total variable cost is the sum of the loss costs (Equation 2) and the reactive cost (Equation 3), or

(4) The cost = $P\,(I_{Q_L} - I_C)^2\,R + C\,I_C$

For the solution of the circuit of Figure I, the cost of losses, P, will be assumed as \$250 per kw and the difference in reactive power cost, C, as \$8 per kva. For these assumed values of \$250, \$8, and a 1000 kva base, the cost of losses, capacitors, and total costs have been plotted as a set of curves in Figure 3. The minimum cost occurs when 440 kva of capacitors are used.

(B) Mathematical Solution

The solution to the circuit of Figure 1 is obtained mathematically as follows. The minimum cost is obtained by taking the derivative of total cost with respect to the capacitor current, setting the derivative equal to zero, and solving for the capacitor currents. This is done as follows:

(4) Total cost = $P\,(I_{Q_L} - I_C)^2\,R + C\,I_C$

$$\frac{d(\text{cost})}{d\,I_C} = 2\,P\,(I_{Q_L} - I_C)\,(-1)\,R + C = 0$$

(5) $I_{Q_L} - I_C = \dfrac{C}{2\,PR} = I_Q$

For the example,

$$I_Q = \frac{8}{(2)\,(250(.1))} = .16$$

The economic reactive current (or the reactive current for minimum cost) is .16 per unit, or 160 kva. The amount of capacitors required to reach minimum cost is 600 - 160, or 440 kva of capacitors. This answer agrees with the minimum cost point of the curves of Figure 3.

(C) Solution by a D-C Circuit

The solution by a d-c circuit is based on Equation 5. Note that Equation 5 has the same form as Ohm's law for a d-c circuit. Equation 5 is in the form of I = E/R when E replaces C/2P. The solution to the system of Figure 1 by a d-c circuit is shown in Figure 4. A d-c voltage, equal to C/2P, forces a current of .16 per unit through a resistor of 0.1 per unit.

Solution of a More Complex Circuit by a D-C Circuit.

If a power system were as simple as that in Figure 1, any method of solution would be easy. To show the value of the d-c circuit method, the system of Figure 5 will be solved. The system consists of one generator, three sections of line, and two loads. The reactive load at bus A is 300 kva and the reactive load at bus B is 300 kva. The line resistances are 6%, 3%, and 6% on a 1000 kva base. Line reactance has been neglected as it is assumed that the load voltage can be held approximately constant by changing the generator voltage. If the load voltage is constant, line reactance does not affect line currents. Again the question to be answered is how many capacitors should be installed at each load bus to give minimum cost.

The solution by a d-c circuit is shown in Figure 6. The generator has been replaced by a d-c voltage equal to C/2P. The two load points have been grounded and the currents measured or calculated. The d-c circuit gives the answer that the reactive currents of 0.133 (or 133 kva) to load A and 0.067 (or 67 kva) to load B will give minimum cost.

The proof that the d-c circuit solution gives the correct answer for the circuit of Figure 5 is given in Appendix I. As shown in Appendix I; the voltage drop to bus A should be C/2P for minimum cost. Thus for any value of current to bus B, there is a current to Bus A for minimum cost. But the real minimum point is reached when the voltage drop to both bus A and bus B is equal to C/2P. The d-c circuit of figure 6 meets this requirement.

Figure 7 shows the total costs of capacitors and I^2R losses due to reactive currents for the circuit of figure 5. There are three curves in figure 7 for three different values of current to Bus A. As the current to Bus B is changed, each curve reaches a minimum cost point. The lowest minimum occurs for the solution as given by the d-c circuit.

Circuits with Many Generators and Loads

The proof of the d-c method for a system with many generators and loads can be handled in a similar manner. Proofs of these more complicated systems have been omitted from this paper. As long as the load voltage is held constant by changing the sending end voltage and the system is a radial type, the method given under section I is an exact solution. If the system contains loop circuits, the difference between the ratio of reactance to resistance (X/R ratio) of the different circuit elements introduces an error. The accuracy of the d-c circuit method for loop circuits is discussed under Section II.

Section II. Capacitor Addition with Constant Receiving and Sending End Voltage

In this section, the sending (or generator) end voltage and the load voltage are both held constant. The exact magnitude of these voltages is not important; the important idea is that these voltages will remain constant. If the load voltage is too low, then capacitors may be more valuable in correcting the voltage drop than in reducing I^2R losses. However, in this section of the paper the load voltage is assumed to be satisfactory and capacitors are installed to reduce I^2R losses.

A simple radial system of one generator, one load, and one line section is shown in Fig. 8. The load voltage is assumed to be 100 per cent. The voltage on the source side of the voltage regulator is lower than 100 per cent, and the voltage regulator is boosting the voltage up to 100 per cent. The current on the source side of the regulator then must be higher than the load current by an amount proportional to the voltage regulator boost.

If a transformer were used in place of the voltage regulator, the principle would be the same. For instance, a transformer stepping down from a 34.5 kv line to a 4.16 kv line, would provide a boost in voltage if the transformer turns ratio was 32 kv to 4.16 kv instead of the nominal 34.5 kv-4.16 kv turns ratio.

Fig. 9 and 10 show the effect of capacitor additions. After capacitors are added, the voltage drop in the line is less and the voltage regulator is not boosting as much as before the capacitor addition. Both the power and reactive current in the line are smaller after the capacitor addition. The power current is smaller by the amount of change in voltage regulator boosting action. Thus capacitors change both the power current and the reactive current.

To solve for minimum cost of capacitors and losses, the procedure used in Section I can be tried. The derivative of total cost will be set equal to zero to find minimum cost.

(6) Total cost = Cost of Losses + Cost of Capacitors

The equation for losses is obtained from Fig. 8, then

(7) $\text{Total cost} = PRN^2 (I_P{}^2 + I_Q{}^2) + C\ I_c$

where

$I_Q = I_{Q_L} - I_C$

I_P = Power Current

N = Ratio of voltage on load side to voltage on source side of voltage regulator (N>1 for voltage regulator to be boosting).

(8) $\dfrac{d(\text{cost})}{d\ I_C} = PRN^2\ 2I_Q(-1) + PR(I_P{}^2 + I_Q{}^2)\ \dfrac{d(N^2)}{d\ I_C} + C = 0$

To solve equation (8), an expression for the change in the amount of voltage boost with capacitor addition (or $d(N^2)/dI_C$) must be found. This is done in appendix II. A quick glance at these expressions, as given in

Appendix II. A quick glance at these expressions, as given in Appendix II, shows that some simplifying approximation must be used. One well known fact can be used to obtain an approximation. This fact is that the voltage change due to capacitor addition is due primarily to the reactance of a line; line resistance has little effect. Using this fact, and letting Es and Er be equal to unit, the following approximation is suggested:

$$(9)\quad \frac{d(N^2)}{D\,I_C} = 2XN^4$$

where

X = per unit reactance

Using Eq. (9), Eq. (8) is set equal to zero and solved for minimum costs. The following equation is obtained:

$$(10)\quad \frac{C}{2\,P} = N_2R\left[I_Q + (I_P{}^2 + I_Q{}^2)\,N^2\,X\right]$$

The d-c circuit representation of this equation is shown on Fig. 11.

To check the accuracy of the approximation of Eq. (9), the simple circuit of Fig. 8 was investigated and the results are shown in Fig. 12. As can be seen on Fig. 12, the approximation gives satisfactory accuracy.

It should be pointed out that when the incremental I^2R loss reduction is equal to the added cost of capacitors, the total cost is at the minimum point. Fig. 12 is plotted on a basis of incremental loss reduction, rather than total cost, in order to provide a check for a wide range of I^2R loss evaluation and reactive costs.

The solution of a circuit by means of Eq. (9) and (10) is still not straightforward. Eq. (10) contains the term "N" which depends on the voltage drop in the line, and thus on the amount of capacitors. This can be handled by choosing an approximate value for "N" and holding it constant. The change in "N" is only a few percent, as from 1.10 to 1.05; so the error is small.

In most cases a further approximation can be made. The equation for the d-c circuit may be written as:

$$(11)\quad \frac{C}{2P} = N^2R\,(I_Q + I_P{}^2\,X)$$

The accuracy of this simpler expression, Eq. 11, can be seen by comparison with Eq. 10. $I_Q{}^2$ is small compared to $I_P{}^2$, and $I_Q{}^2$ can be neglected. Dropping the second "N^2 term would usually introduce only a small error. Where Eq. 10 or Eq. 11 should be used, depends on the desired accuracy.

System Voltage Drop Corrected at Several Points

In practice, the voltage drop is not corrected at only one point, but the voltage drop is corrected at several points. A radial representation of a power system is shown in Fig. 13. As capacitors are added at the load point, the voltage is adjusted at three points by changing transformer taps or the voltage regulator. The voltage is assumed to be constant at the capacitor location (EL), and at points E2 and E3 on the subtransmission system and the transmission system.

In Fig. 14, a d-c circuit is given for determining the incremental I^2R loss reduction per kva of capacitor. The voltage drop was computed on the basis of unity power factor operation and this voltage drop was used in determining "N". Fig. 14 gives an example of calculating the I^2R loss reduction when the reactive load current is 0.2 per unit. The calculation could, of course, be reversed and the reactive current for minimum cost found by applying a voltage of C/2P and measuring or calculating the reactive current, I_Q.

The voltage drop has been computed at unity power factor for two reasons. One is that unity power factor means that the voltage drop can easily be computed. Also, if the term "N" is computed for unity power factor, then the d-c circuit method will under-estimate the I^2R loss reduction for any power factor below unity.

Fig. 15 shows a comparison of the exact calculation by means of Eq. (7) and (8), an approximate method when the exact voltage drop is known, Eq. (9) and (10), and the approximate method of Eq. (11) and Fig. 14. In the region of assumed interest (the voltage drop was computed for 1.0 P.F. operation), the three methods check very closely.

For purposes of comparison, a curve is shown with the I^2R loss reduction calculated on the basis of the method of Section I.

In changing transformer taps to maintain the same voltage as before a capacitor addition, it was assumed that the transformer taps could be changed by a very small amount. If changes of the transformer taps were made in definite steps, as 2½% tap changes, the curves would have the same general slope but would have a saw-tooth shape.

Errors Introduced by Loop Circuits

The preceeding part of the paper has assumed that power systems are radial systems. On loop systems, the d-c circuit method will have about the same accuracy as on a radial sysem if the X/R ratio of all sections of the loops are the same. However, this is not the usual case.

Transmission and subtransmission loops will have line sections with different X/R ratios due to the different conductor sizes. The difference in X/R ratios, or conductor sizes, is usually less than two or three to one. To obtain an idea of the magnitude of this error; a simple system was investigated, one generator feeds one load over two parallel lines. The reduction of I^2R losses were computed by the d-c circuit and by a more exact longhand method. The results are shown in Fig. 16. For most cases, the line impedances would differ in X/R ratios, by a factor of less than 2:1, the error for this loop would be less than 10%. If the X/R ratios differ by a factor of 5:1, the error in the loop portion is high. A redemming factor is that the losses in such loop portions of a system are usually a minor part of the total loss; so the overall accuracy would be much better.

There are many types of loops and Fig. 16 is therefore only an indication of the magnitude of the error due to loop circuits. More work is needed to determine the accuracy of the d-c system for more complicated loop systems.

Section III. Mechanics of D-C Circuit Solution

A power system is too complicated to individually represent the resistance of all transformers and lines in the transmission, subtransmission, and distribution systems at the same time. Of necessity, some simplification must be made; the simplification should, however, represent the total loss. One possible method would be to represent one section of the power system in detail and then use equivalent lumped resistances and loads for the other parts of the system. Reference 6 can be used to determine average distribution primary losses.

Fig. 17 shows the one line diagram of the transmission section of a power system. The subtransmission and distribution losses have been represented as concentrated loads and resistances. As capacitors are added, transformer taps and voltage regulator hold constant voltage at all points marked "X". The d-c circuit for the solution of this system is shown on Fig. 18.

When a solution has been obtained for the transmission system, the division of reactive current between the different distribution substations and feeders can be found from a circuit, as shown in Fig. 19. The losses in the transmission and generation sections have been represented by concentrated or an equivalent loss.

If the economical reactive current to this section of the system were known, as from a solution similar to that shown in Fig. 18, the "R system" of Fig. 19 would be adjusted to permit the economical amount of reactive current to flow to this part of the system. Fig. 19 would then be used to determine the division of reactive current between the substations and feeders in this area.

An alternate method would be to skip a system solution, as represented in Fig. 18, and to only obtain a solution by means similar to Fig. 19. "R system" would then be determined from known losses in the generation and transmission area. "R system" would then be an average system loss to the subtransmission or distribution area. This alternate solution would be approximate in that it assumes the same generation and transmission loss to each area.

Loads Near the Generators

Loads that are near the generator, for instance, those fed from the generator bus, may not justify any capacitors on the basis of reduced I^2R losses. The d-c circuit method, as presented this far, is blind to the reactive currents now existing in the system. A given load may have a power factor of 80 per cent. A d-c circuit solution may give an answer that the economical power factor is only 70%. This, of course, means that no capacitors should be installed at that load point. To obtain the true effect of capacitors in reducing I^2R losses for the other loads of the system; the reactive current to this one load must be limited to its present reactive current, i.e. that corresponding to 80% P.F. This can be done in a d-c circuit by adding sufficient resistance in series with this load point to limit the current to the present reactive current.

Generators Near Load

The same situation may occur for one generator that is nearer the load than are the other generators. In such a case the d-c circuit solution may say that one generator should supply reactive current beyond the capability of the generator. Here also, a resistor must be added in series with the generator to limit the current to the capability of the generator.

Synchronous Condenser and Line Charging

Synchronous condensers and transmission line charging may be represented as kilovar generators with a constant kilovar output. This representation would be similar to that of a generator located near the load. Whether line charging should be represented depends upon the magnitude of the line charging and the desired accuracy.

Leading Power Factor Loads

Fig. 15 shows that the power factor of some loads may be economically raised into the leading power factor region. The question of how to handle such loads may arise. The answer can be found by referring to Fig. 14. In Fig. 14, the total current flowing in the distribution area is "$I_Q + 0.13$". If a d-c circuit solution gave an answer that the total current in the distribution area should be 0.09, this would mean that the reactive current should be 0.04 lead (.09-.13).

Generator Losses

The generator loss in Fig. 13 has been given as 0.75%. As 0.75% loss is higher than the usual generator armature I^2R loss, an explanation is in order.

The 0.75% resistor represents the change in generator losses as the power factor changes. The 0.75% resistor does not represent just the armature I^2R loss; it represents the change in armature I^2R loss, field loss, load loss, and windage and friction loss. For example, assume that a generator operating at 66,000 kw and unity power factor has a total loss of 800 kw. At 66,000 kw, 0.85% P.F., and 77,650 kva, the total loss might be 960 kw. A resistor, to give the same change in loss when the kva changes from 77,650 kva to 66,000, is found as follows:

$$\text{Change in Loss} = 960 - 800 = 160 \text{ kw}$$

$$\text{Per cent change in loss on 77,650 kva base} = \frac{160}{77{,}650} \times 100 = .208\%.$$

$$\text{Per cent change in kva on 77,650 kva base} = \frac{77{,}650 - 66{,}000}{77{,}650} \times 100 = 15\%$$

Percent R will give 0.208% change in loss when kva changes

$$\text{from } 100\% \text{ to } 85\% = \frac{.208}{1.00 - .85^2} = \frac{.208}{1.00 - .723} = 0.75\%$$

D-c Board Base

When the voltages (C/2P) and the per unit resistances are expressed as numbers, they are all very small numbers. However, when the voltage and per unit resistance are both multiplied by the same number, the current remains in per unit. Thus, to obtain numbers that can be conveniently handled on a d-c board, all voltages and resistances can be multiplied by 10, 100, or 1000--or by any number.

Kw of Loss May Have Different Value in Different Parts of the System

A Kw of I^2R loss may be evaluated differently in different parts of the system. For instance, a kw of loss may be evaluated on the basis of $350 in the distribution system, $300 in the transmission system, and $250 at the generator. This difference would be due to the evaluation of the cost of the system required to transfer a kw from the generator to the transmission and distribution areas.

This difference in the value of a kw of loss (or P) raises the question of what value should be used in determining the d-c voltage of C/2P.

This problem may be handled by choosing one value, say $250, and then using that value to determine the d-c voltage of C/2P. The resistances in the transmission area should then be increased by the ratio of $300 to $250, and the resistances in the distribution area should be increased by the ratio of $350 to $250. This procedure of using artificial values of resistances would then give the correct value of I^2R loss reduction in terms of dollars per kva of capacitor.

List of Symbols

C = Difference in cost of a kva of reactive power supplied at the load and a kva of reactive power supplied by a generator.

C_G = Cost of a kva of reactive power supplied from the generator.

C_L = Cost of a kva of reactive power supplied at the load by capacitors.

E_R = Load or receiving end voltage.

E_S = Generator or sending end voltage.

I_A, I_B = Reactive current flowing to bus A and B

I_{AC}, I_{BC} = Capacitor current at bus A and B.

I_{AL}, I_{BL} = Reactive current of load at bus A and B

I_C = Capacitor current at load point

I_Q = Reactive current flowing to the load point

I_{QL} = Reactive current of the load

I_P = Power current at the load

N = Ratio of voltage on load side to voltage on source side of voltage regulator ($N > 1$ for voltage regulator to be boosting).

P = Dollar value of a kw of I^2R loss.

R = Line resistance

X/R = Ratio of reactance to resistance

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6) "Useful Methods for Determining Primary Feed Points in Future Distribution System Planning", D. L. Hopkins, D. R. Samson, Vol. 73, Part III-B, 1954, pp. 856-863.

APPENDIX I

The proof that the d-c circuit solution gives the correct answer for the circuit of Figure 5 is as follows:

Only reactive currents will be considered.

(1) Cost of I^2R loss $= L = P\ (I_{AL}-I_{AC})^2\ R_2 + P\ (I_{BL}-I_{BC})^2\ R_3 + P\ (I_{AL}-I_{AC} + I_{BL}-I_{BC})^2\ R_1$

(2) Cost of capacitors $= K = C\ I_{AC} + C\ I_{BC}$

I_{AL}, I_{BL} = reactive current of loads A and B

I_{AC}, I_{BC} = capacitor currents at bus A and B

$I_A, \ I_B$ = reactive current flowing to bus A and B

(3) Total cost $= L + K$

The minimum cost is obtained by setting the derivative of Equation 3 equal to zero. For changing capacitor current at bus A and constant capacitor current at bus B:

(4) $$\frac{d(\text{cost})}{d\ I_{AC}} = 2\ P\ R_2\ (I_{AL}-I_{AC})\ (-1) + PR_1\ (I_{AL}-I_{AC} + I_{BL}-I_{BC})\ (-1) + C = 0$$

$$(I_{AL}-I_{AC})\ R_2 + (I_{AL}-I_{AC} + I_{BL}-I_{BC})\ R_1 = C/2P$$

(5) $I_A \cdot R + (I_A + I_B)\ R_1 = C/2P$

For any value of capacitor current at bus B, Equation 5 will determine the value of capacitors at bus A to give minimum cost. The term $I_A \cdot R_2$ is the voltage drop in the d-c circuit in R_2. The term $(I_A + I_B)\ R_1$ is the voltage drop in the dpc circuit in R_1. The sum of these two terms is the voltage drop in a d-c circuit from the battery to the load at A.

When the derivative of Equation 3 is taken with the capacitor current at bus B changing (and capacitor current at load A held constant, the resulting equation which gives the value of capacitor current at bus B for minimum cost:

$$(I_{BL}-I_{AC})\ R_3 + (I_{AL}-I_{AC} + I_{BL}-I_{BC})\ R_1 = C/2P$$

(6) $I_B\ R_3 + (I_A + I_B) R_1 = C/2P$

Equation 6 says that the voltage drop to bus B must be equal to C/2P in order for cost to be minimum; Equation 5 also said that the voltage drop to bus A be equal to C/2P in order for cost to be minimum. As these two conditions are met by the d-c circuit of Figure 6, the d-c circuit must give the correct answer.

APPENDIX II

The rate of change of the amount of voltage boost with capacitor addition is found as follows:

From Fig. 8

(1) $E_S = \frac{E_R}{N} + NI_PR + NI_QX + j(NI_PX-NI_QR)$

Squaring both sides of equation 1 and solving as a quadratic equation, the following expression is found for the turns ratio:

(2) $$\frac{1}{N^2} = \frac{E_S^2}{2E_R^2} - \frac{I_PR}{E_R} - \frac{I_QX}{E_R} + \sqrt{\frac{E_S^4}{4E_R^4} + \frac{2\ I_PI_QRX}{E_R^2} - \frac{E_S^2}{E_R^3}\ I_PR - \frac{E_S^2}{E_R^3}\ I_QX - \frac{I_P^2X^2}{E_R^2} - \frac{I_Q^2\ R^2}{E_R^2}}$$

The derivative of N^2 is taken and the following equation results:

(3) $$\frac{d(N^2)}{dI_C} = \cfrac{\frac{-X}{E_R} - \left[\frac{E_S^2X}{2E_R^3} + \frac{I_QR^2}{E_R^2} - \frac{I_PRX}{E_R^2}\right]}{\cfrac{\sqrt{\frac{E_S^4}{4E_R^4} + \frac{2I_PI_Q\ RX}{E_R^2} - \frac{E_S^2}{E_R^3}\ I_PR - \frac{E_S^2I_QX}{E_R^3} - \frac{I_P^2X^2}{E_R^2} - \frac{I_Q^2\ R^2}{E_R^2}}}{\frac{1}{N^4}}}$$

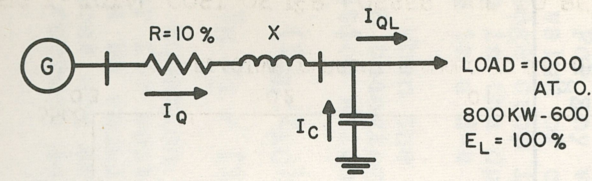


FIG. 1-ONE LINE DIAGRAM OF SIMPLE SYSTEM

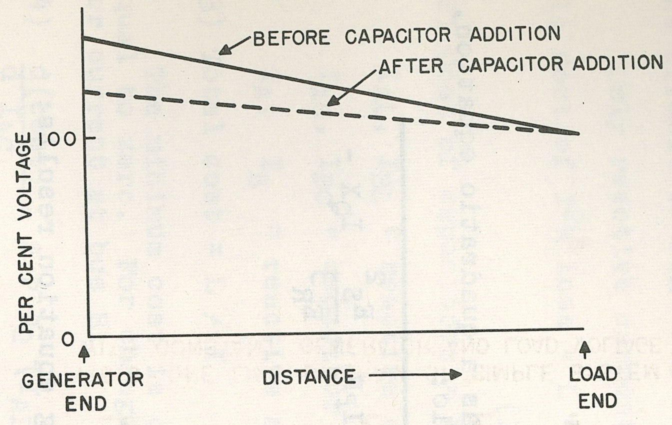


FIG. 2-VOLTAGE PROFILE OF SIMPLE SYSTEM OF FIG. 1

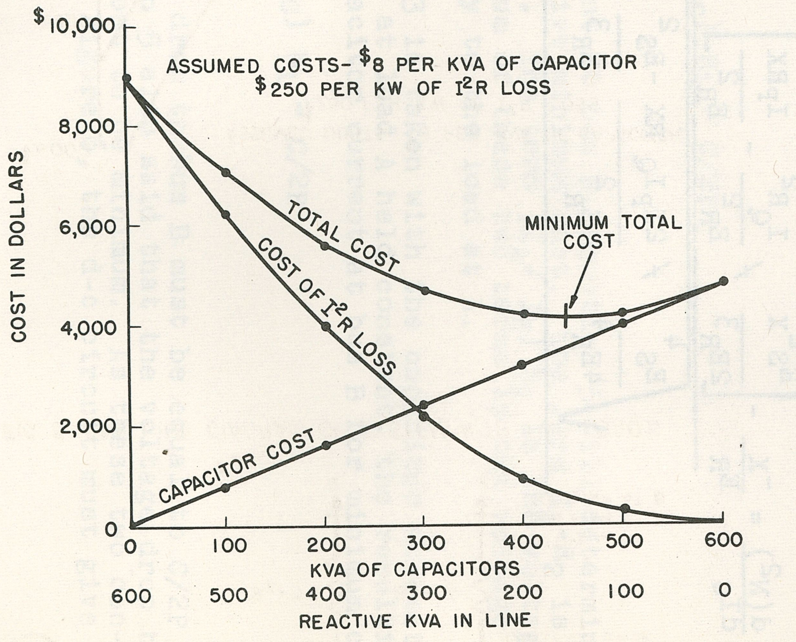
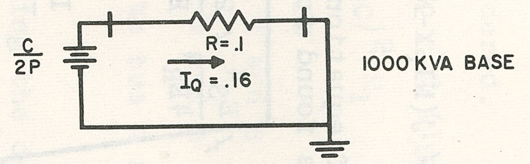


FIG. 3 COST OF I^2R LOSSES DUE TO REACTIVE CURRENT AND COST OF CAPACITORS FOR SYSTEM OF FIG.1



$I_Q = \frac{\frac{C}{2P}}{R} = \frac{8}{(2)(250)(.1)} = 0.16$

$I_C = (.6-.16)(1000) = 440$ CKVA

FIG. 4 D-C CIRCUIT SOLUTION FOR MINIMUM COST FOR SIMPLE SYSTEM OF FIG. 1

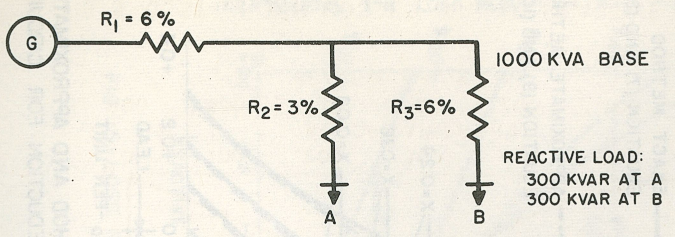
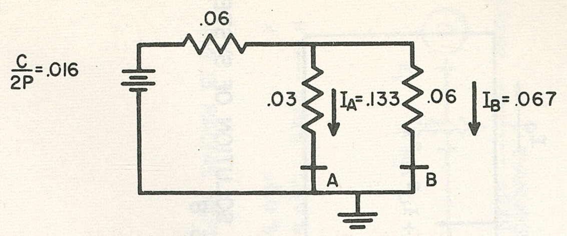


FIG. 5 ONE LINE DIAGRAM OF SYSTEM WITH TWO LOADS



I_{AC} =(.30-.133)(1000)= 167CKVA

I_{BC} =(.30-.067)(1000)=233CKVA

FIG. 6-D-C CIRCUIT SOLUTION FOR MINIMUM COST FOR SYSTEM OF FIG.5

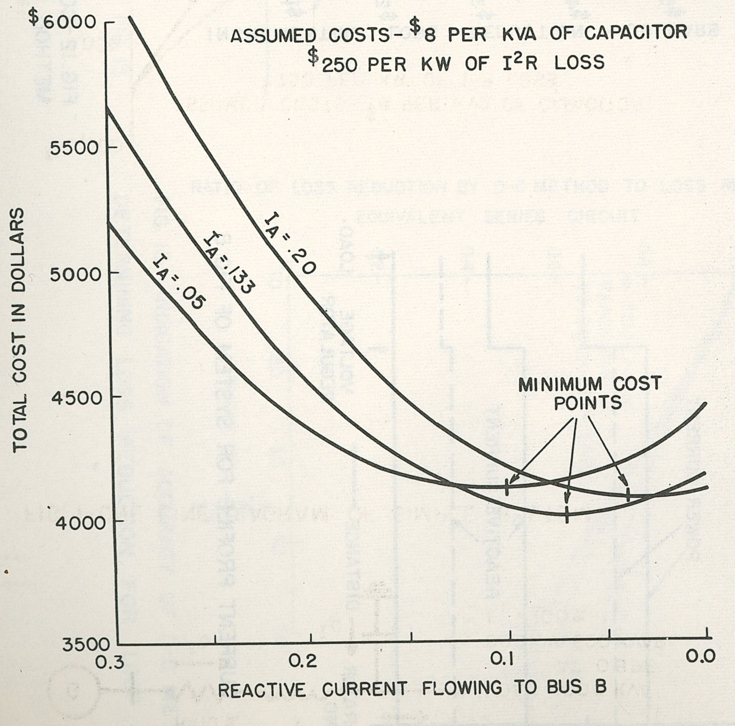


FIG. 7-TOTAL COST OF I^2R LOSSES DUE TO REACTIVE CURRENT AND COST OF CAPACITORS FOR SYSTEM OF FIG.5

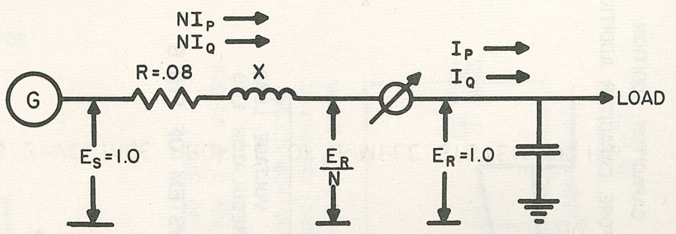


FIG. 8-ONE LINE DIAGRAM OF SIMPLE SYSTEM WITH CONSTANT GENERATOR AND LOAD VOLTAGE

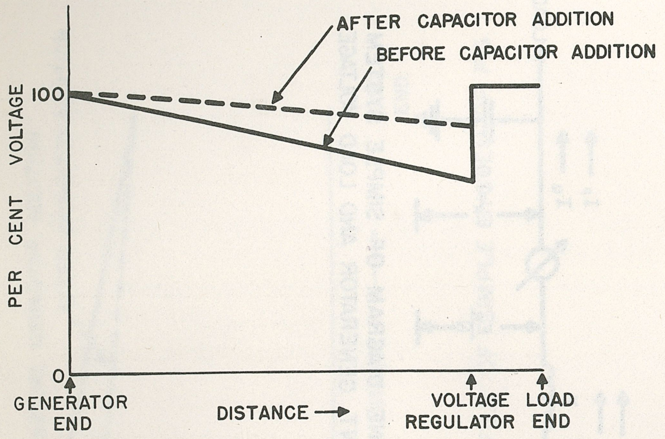


FIG. 9-VOLTAGE PROFILE FOR SYSTEM OF FIG. 8

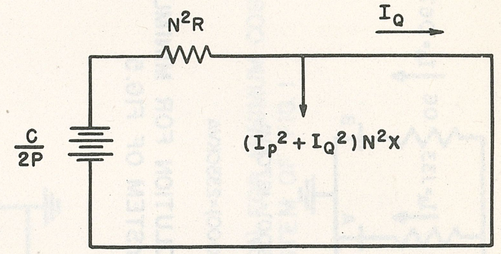


FIG. 11 D-C CIRCUIT FOR SOLUTION OF SYSTEM OF FIG. 8

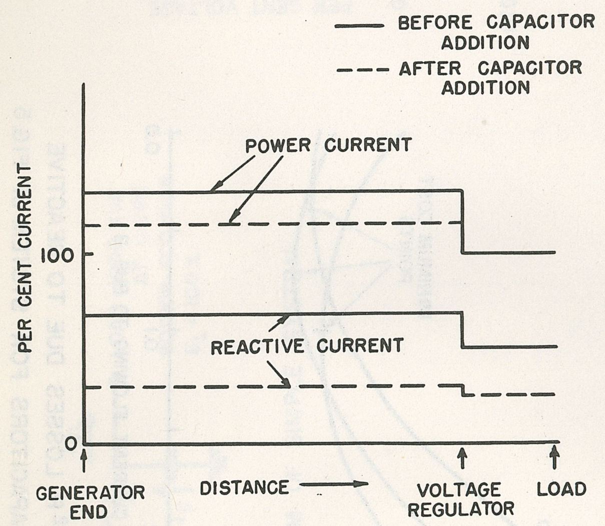


FIG. 10 CURRENT PROFILE FOR SYSTEM OF FIG. 8

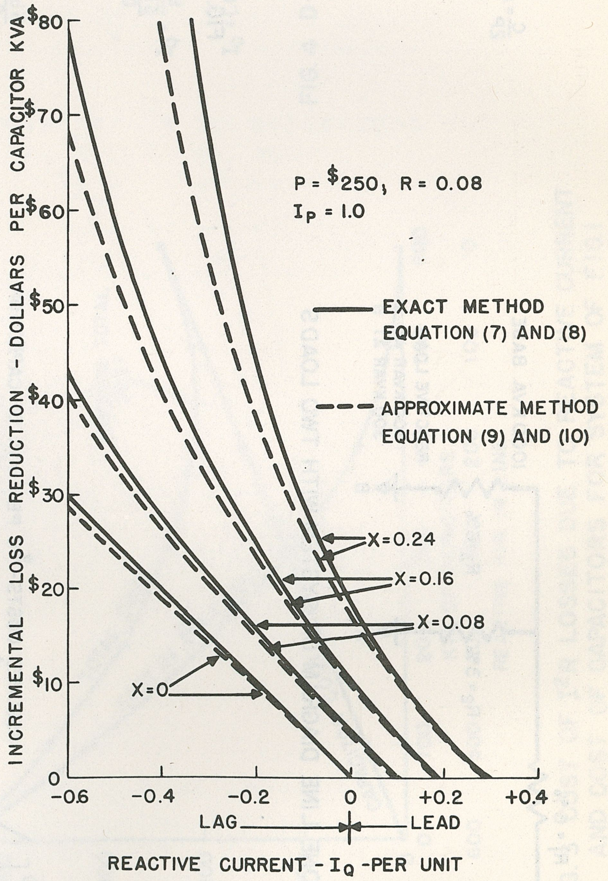


FIG. 12 - COMPARISON OF EXACT METHOD AND APPROXIMATE METHOD FOR DETERMINING LOSS REDUCTION FOR CIRCUIT OF FIG. 8

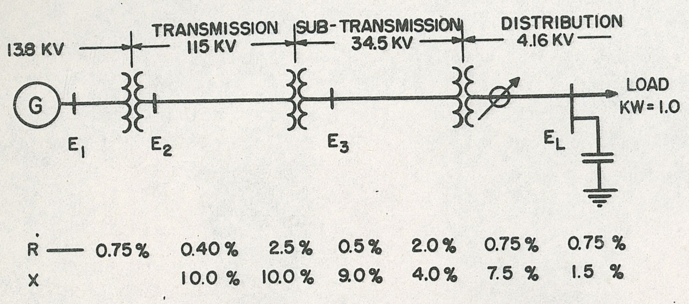
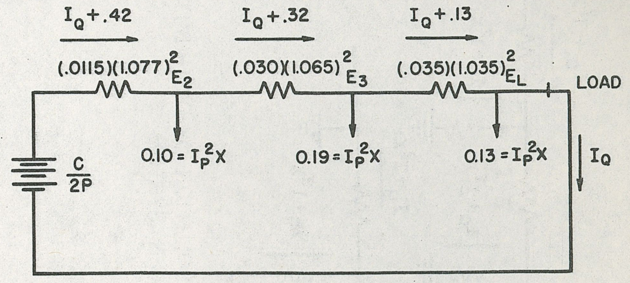


FIG. 13 RADIAL REPRESENTATION OF A POWER SYSTEM



WHEN $I_Q = 0.20$ AND P = $250 PER KW
C = (2)(250) [(.62)(.0133) + (.52)(.0340) + (.33)(.0375)] =
$19.00 PER CKVA

FIG. 14 D-C SOLUTION FOR CIRCUIT OF FIG. 13 BY USE OF APPROXIMATE EQUATION (11)

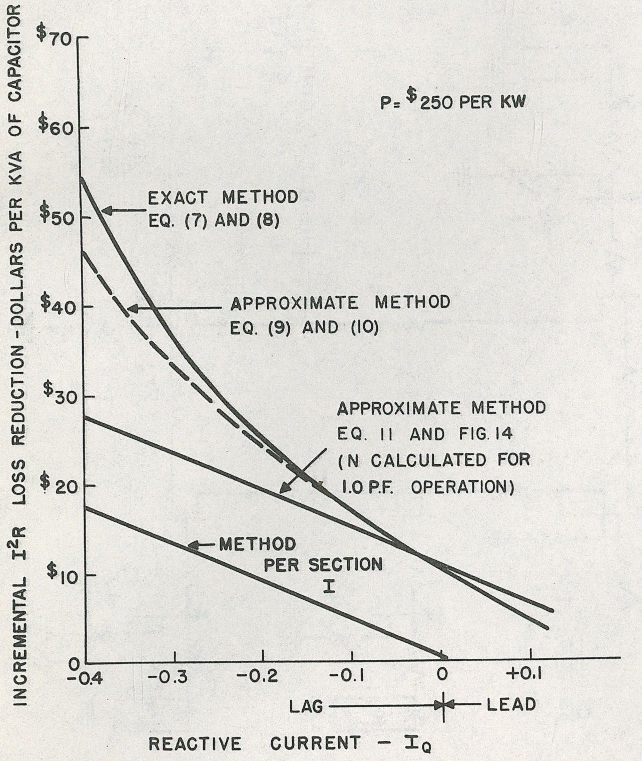


FIG. 15 - LOSS REDUCTION FOR CIRCUIT OF FIG. 13

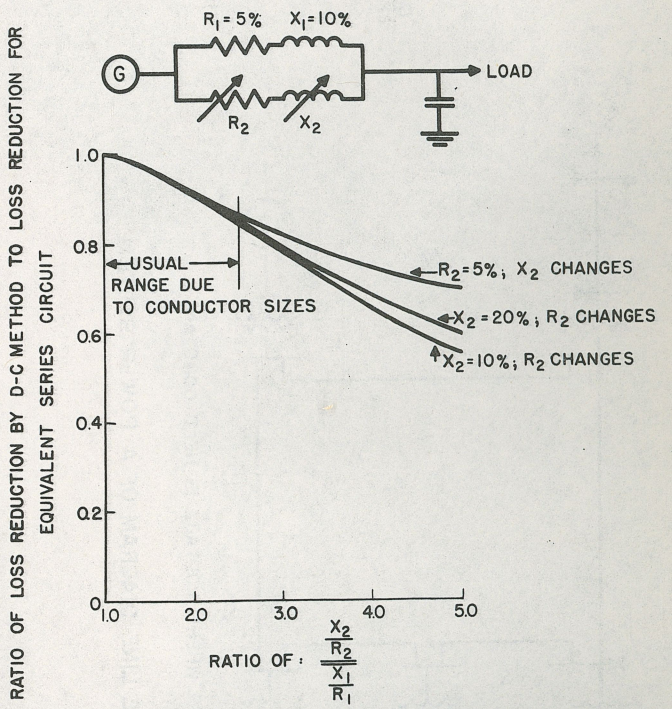


FIG. 16 - INDICATION OF ACCURACY OF D-C METHOD OF DETERMINING LOSS REDUCTION FOR LOOP CIRCUITS

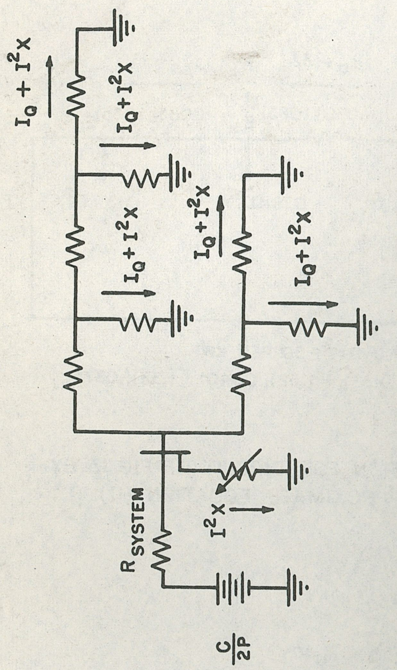


FIG. 19 D-C CIRCUIT FOR SOLUTION OF DISTRIBUTION PORTION OF POWER SYSTEM

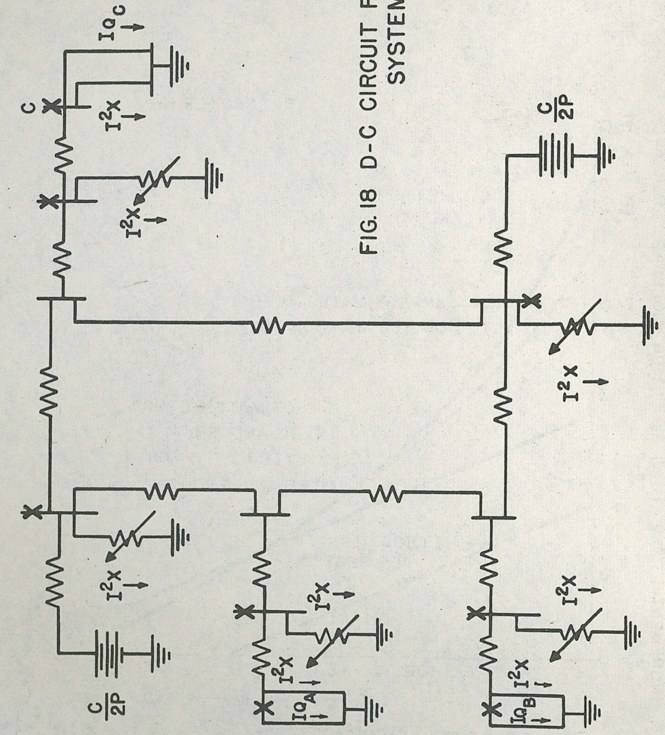


FIG. 18 D-C CIRCUIT FOR SOLUTION OF A POWER SYSTEM OF FIG. 17

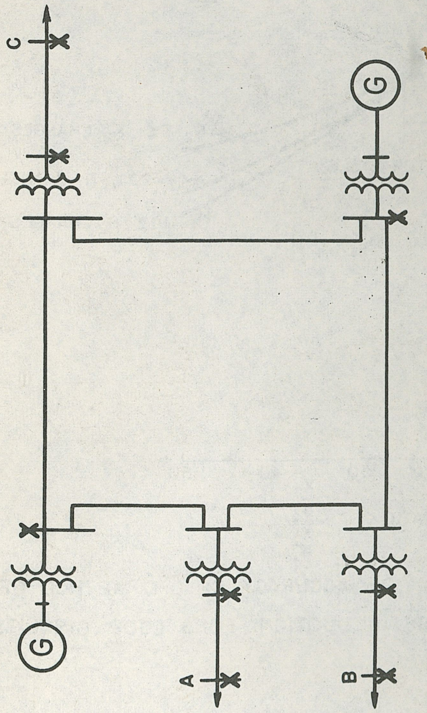


FIG. 17 ONE LINE DIAGRAM OF A POWER SYSTEM

X- POINTS WHERE VOLTAGE IS HELD CONSTANT

Mr Kingsbury

Conference Paper

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D-C CIRCUIT GIVES EASY METHOD OF DETERMINING VALUE OF CAPACITORS IN REDUCING I^2R LOSSES

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